# Ruminations on the Normalcy of Stocks

There has been a great deal of discussion over the past few years about the stock market exhibiting "fat tails" where the recent downturn was considered to be an outlier. I thought it might be useful to investigate just how "normal" (or not) the market really is. Normal means the distribution of returns matches the standard bell curve.

**Some Statistics.** To describe a distribution, first we need to know where it is centered. In statistics this is called the first moment (because the formula that computes it only has terms raised to the first power, i.e. nothing is raised to anything, it is just a simple arithmetic mean). The second moment (because the formula contains squared terms) is the standard deviation or sigma ($\sigma$). This is a measure of how spread out the data is; and it is the square root of the variance. Those two statistics are relatively familiar, but the normality of the data is dependent on what are called the higher moments.

The third moment is skewness, a measure of whether the distribution is symmetrical or not. If not, it has skew. (The formula, as you have no doubt realized by now, has cubed terms.) In a normal distribution, the mean and median (and mode for that matter) are identical. In a skewed distribution they are not. Positively skewed distributions have a tail going out further on the right, with the mean being higher than the median. A negatively skewed distribution has a longer tail on the left with the mean being lower than the median.

An example of positive skew would be a graph of the distribution of wealth for a random group of people that happened to include Bill Gates; on *average* they are all very wealthy. Life expectancies, on the other hand, exhibit negative skew – it is much easier to die 50 years before your life expectancy than it is 50 years after. In investing negative skew is unfortunate because the investor receives more extreme negative results than extreme positive results. In theory, investment returns should have a slight positive skew due to compounding, and (again in theory) would follow what is known as a log-normal distribution, which simply means the logarithms of the returns follow a normal distribution.

The fourth moment is kurtosis, and this is a measure of the "peakedness" of the distribution. A distribution with a high middle but more weight in the tails can have the same average dispersion (standard deviation) as a distribution with the opposite. When you hear the term "fat tails" or "black swan," usually what the person really means is positive kurtosis. A normal curve is said to be mesokurtic; while one with fat tails and a higher peak is leptokurtic; and one with skinny tails (or no tails) and a lower peak is platykurtic. It is nice to know if your investments will have more extreme events than you would expect (i.e. is leptokurtic), particularly if they are negatively skewed as well. The remainder of this paper is devoted to examining empirically what the distribution of returns actually is.

**Daily Data.** The length of the period we are measuring is important. There is no question that at time frames as short as one day the market is not even close to normal. For example, if, on October $18^{th}$, 1987 (the day before the famous collapse) you had computed the statistics on the S&P 500 since 1950, you would have found that the odds of a decline as big as the one that actually took place the following day were astronomically (actually bigger than astronomically, but I can't think of an appropriate adverb) against it. It was a ***26*** standard deviation event.

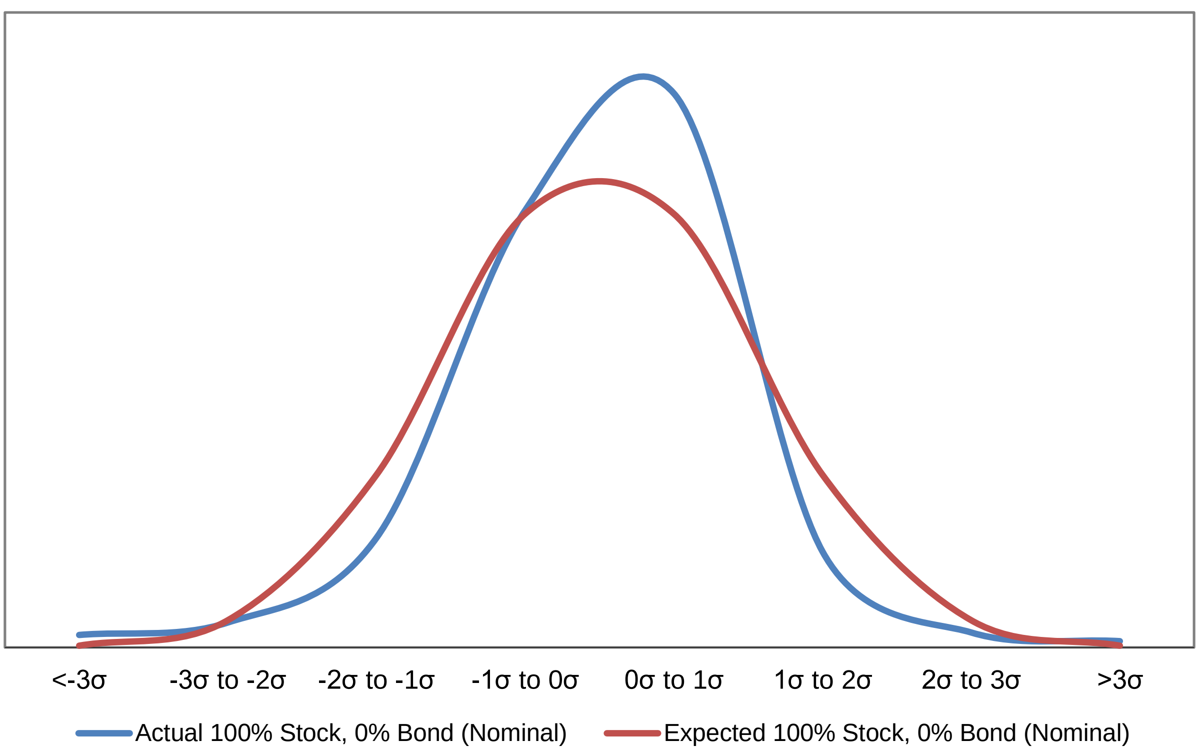
Some context about how often daily moves in the stock market of various sizes should be expected (if they were normally distributed) is helpful here:

| Standard Deviation | Expected Occurance |
|---|---|
| One | every three days or so |
| Two | every month or so |
| Three | every 1½ years or so |
| Four | every 63 years or so |
| Five | every 7 millennia or so |
| Six | every 2 million years or so |
| Seven | every 1½ billion years or so |
| Eight | every 236 times the age of the universe |
| Nine and higher | the numbers are too big to quantify |

Keep in mind the 1987 crash was a ***26*** standard deviation event. So markets on a daily basis clearly are not normally distributed and have extreme outliers, and extreme daily down moves are bigger than daily up moves (but there are slightly more up overall) as well. So we can unequivocally state that short term market moves (daily or less) are negatively skewed and leptokurtic (and extremely so on both counts).

**Monthly Data.** Monthly stock market moves are closer to normal. The stock data used here is the total U.S. stock market (CRSP 1-10) from 1926 through 2010. I will show just the nominal, normal graphs. The real (inflation adjusted) and log-normal graphs are almost identical. Here is a graph of the distribution of nominal returns over the 1,020 months in our sample:

**Distribution of Monthly Returns 1926 - 2010**



On a monthly basis we can conclude there is mild skew (positive compared to a normal distribution and negative compared to a log-normal distribution) and significant positive kurtosis. Here are the specific numbers:

| | Nominal Stocks | | Real Stocks | |
|---|---|---|---|---|
| | **Monthly** | **Monthly LN** | **Monthly** | **Monthly LN** |
| Arithmetic Average: | 0.91% | 0.76% | 0.66% | 0.51% |
| Sigma: | 5.42% | 5.42% | 5.44% | 5.44% |
| Geometric Average: | 0.76% | 0.62% | 0.52% | 0.37% |
| Skewness: | 0.145 | -0.546 | 0.254 | -0.438 |
| Significant? | No (1.89<2) | Yes (7.12>2) | Yes (3.31>2) | Yes (5.71>2) |
| Kurtosis: | 7.323 | 6.515 | 7.501 | 6.305 |
| Significant? | Yes (47.74>2) | Yes (42.47>2) | Yes (48.9>2) | Yes (41.11>2) |
| Maximum: | 37.90% | 32.14% | 38.28% | 32.41% |
| Minimum: | -28.67% | -33.79% | -28.36% | -33.35% |
| # of Observations: | 1020 | 1020 | 1020 | 1020 |

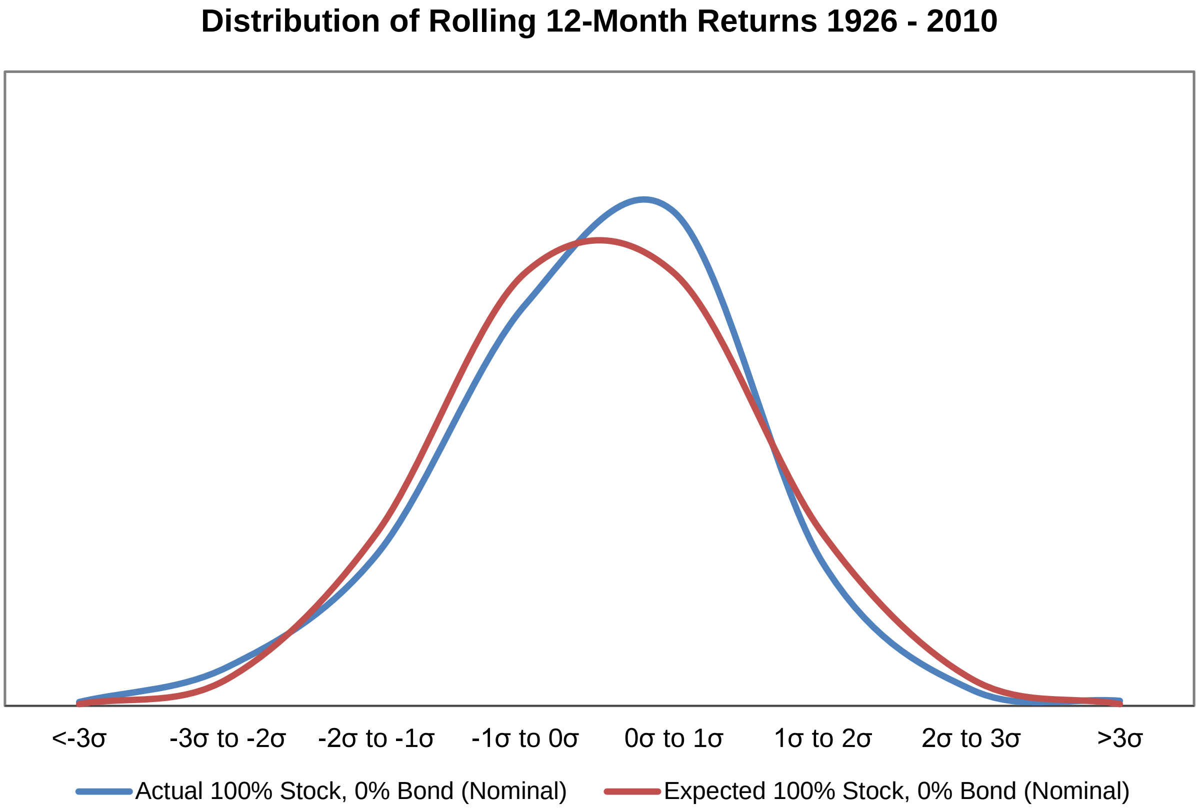
And here is how it broken down by standard deviation:

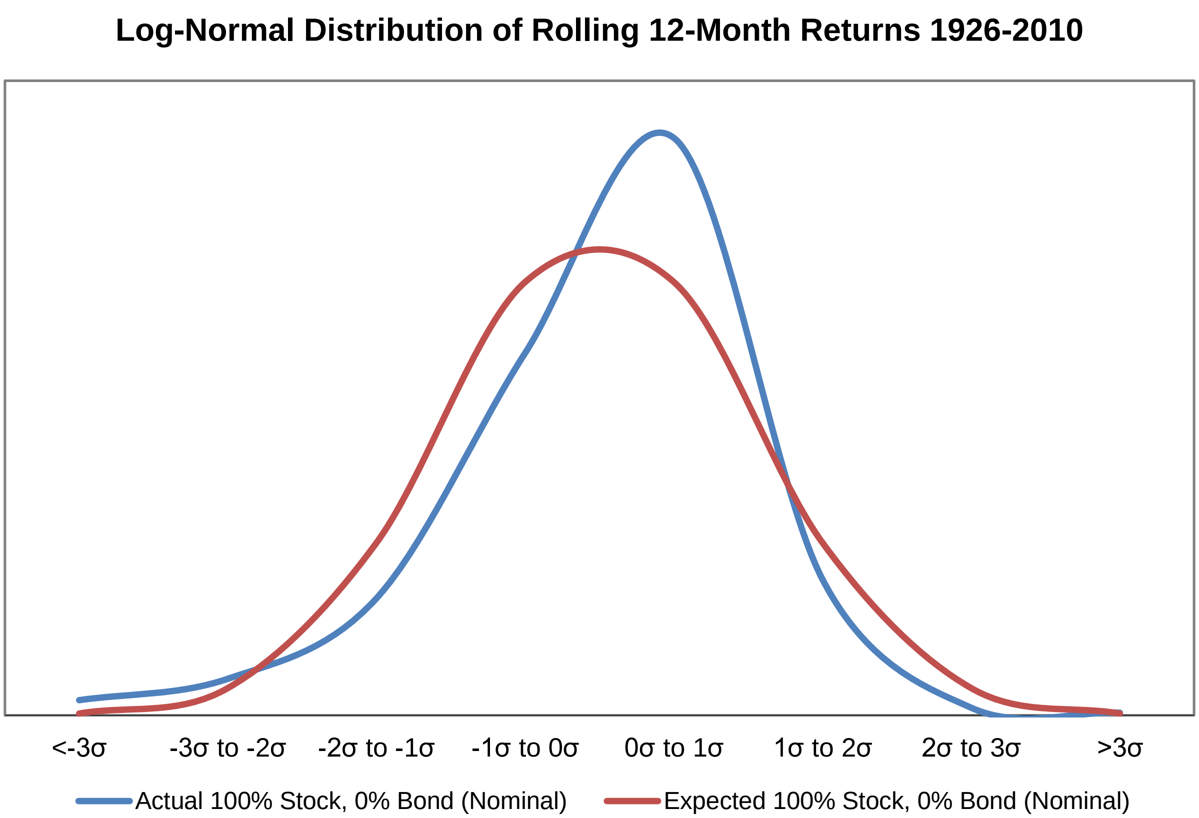
| Monthly (Nominal) Stock Returns | | | | | | |
|---|---|---|---|---|---|---|
| **σ** | **Actual** | **%** | **Expected** | **%** | **Difference** | **%** |
| <-3σ | 10 | 0.98% | 1 | 0.13% | 9 | 0.85% |
| -3σ to -2σ | 20 | 1.96% | 22 | 2.14% | -2 | -0.18% |
| -2σ to -1σ | 88 | 8.63% | 139 | 13.59% | -51 | -5.02% |
| -1σ to 0σ | 351 | 34.41% | 348 | 34.13% | 3 | 0.28% |
| 0σ to 1σ | 445 | 43.63% | 348 | 34.13% | 97 | 9.60% |
| 1σ to 2σ | 77 | 7.55% | 139 | 13.59% | -62 | -6.11% |
| 2σ to 3σ | 12 | 1.18% | 22 | 2.14% | -10 | -0.97% |
| >3σ | 5 | 0.49% | 1 | 0.13% | 4 | 0.36% |
| >avg | 539 | 52.84% | 510 | 50.55% | 29 | 2.87% |
| <avg | 469 | 45.98% | 510 | 50.55% | -41 | -4.06% |

| Monthly Log-Normal (Nominal) Stock Returns | | | | | | |
|---|---|---|---|---|---|---|
| **σ** | **Actual** | **%** | **Expected** | **%** | **Difference** | **%** |
| <-3σ | 11 | 1.08% | 1 | 0.13% | 10 | 0.95% |
| -3σ to -2σ | 17 | 1.67% | 22 | 2.14% | -5 | -0.48% |
| -2σ to -1σ | 81 | 7.94% | 139 | 13.59% | -58 | -5.71% |
| -1σ to 0σ | 339 | 33.24% | 348 | 34.13% | -9 | -0.91% |
| 0σ to 1σ | 451 | 44.22% | 348 | 34.13% | 103 | 10.19% |
| 1σ to 2σ | 74 | 7.25% | 139 | 13.59% | -65 | -6.40% |
| 2σ to 3σ | 10 | 0.98% | 22 | 2.14% | -12 | -1.17% |
| >3σ | 5 | 0.49% | 1 | 0.13% | 4 | 0.36% |
| >avg | 551 | 54.02% | 510 | 50.55% | 41 | 4.06% |
| <avg | 457 | 44.80% | 510 | 50.55% | -53 | -5.25% |

Phone: 770-517-8160 • Fax: 770-517-8159 • Toll Free: 888-Fin-Arch (888-346-2724)
107 Weatherstone Drive, Suite 510 • Woodstock, GA 30188
www.FinancialArchitectsLLC.com

If we lengthen the period to the 1,009 rolling 12-month periods we see the fit is much better with a normal distribution, but oddly, the fit with a log-normal distribution actually appears *worse*:



Here are the specific numbers:

| | Nominal Stocks | | Real Stocks | |
|---|---|---|---|---|
| | 12-Month | 12-Month LN | 12-Month | 12-Month LN |
| Arithmetic Average: | 11.69% | 9.03% | 8.41% | 6.02% |
| Sigma: | 21.65% | 20.77% | 21.69% | 20.66% |
| Geometric Average: | 9.35% | 6.87% | 6.06% | 3.89% |
| Skewness: | 0.187 | -1.027 | 0.600 | -0.629 |
| Significant? | Yes (2.42>2) | Yes (13.32>2) | Yes (7.78>2) | Yes (8.15>2) |
| Kurtosis: | 2.983 | 3.051 | 4.631 | 1.856 |
| Significant? | Yes (19.34>2) | Yes (19.78>2) | Yes (30.03>2) | Yes (12.03>2) |
| Maximum: | 154.84% | 93.55% | 172.91% | 100.40% |
| Minimum: | -65.41% | -106.16% | -61.60% | -95.71% |
| # of Observations: | 1009 | 1009 | 1009 | 1009 |

And again, here is how it broken down by standard deviation:

| Rolling 12 Month (Nominal) Stock Returns | | | | | | |
|---|---|---|---|---|---|---|
| σ | Actual | % | Expected | % | Difference | % |
| <-3σ | 3 | 0.30% | 1 | 0.13% | 2 | 0.16% |
| -3σ to -2σ | 31 | 3.07% | 22 | 2.14% | 9 | 0.93% |
| -2σ to -1σ | 120 | 11.89% | 137 | 13.59% | -17 | -1.70% |
| -1σ to 0σ | 319 | 31.62% | 344 | 34.13% | -25 | -2.52% |
| 0σ to 1σ | 393 | 38.95% | 344 | 34.13% | 49 | 4.81% |
| 1σ to 2σ | 114 | 11.30% | 137 | 13.59% | -23 | -2.29% |
| 2σ to 3σ | 13 | 1.29% | 22 | 2.14% | -9 | -0.85% |
| >3σ | 4 | 0.40% | 1 | 0.13% | 3 | 0.26% |
| >avg | 524 | 51.93% | 505 | 50.00% | 20 | 1.93% |
| <avg | 473 | 46.88% | 505 | 50.00% | -32 | -3.12% |

| Rolling 12 Month Log-Normal (Nominal) Stock Returns | | | | | | |
|---|---|---|---|---|---|---|
| σ | Actual | % | Expected | % | Difference | % |
| <-3σ | 12 | 1.19% | 1 | 0.13% | 11 | 1.05% |
| -3σ to -2σ | 29 | 2.87% | 22 | 2.14% | 7 | 0.73% |
| -2σ to -1σ | 93 | 9.22% | 137 | 13.59% | -44 | -4.37% |
| -1σ to 0σ | 288 | 28.54% | 344 | 34.13% | -56 | -5.59% |
| 0σ to 1σ | 459 | 45.49% | 344 | 34.13% | 115 | 11.36% |
| 1σ to 2σ | 108 | 10.70% | 137 | 13.59% | -29 | -2.89% |
| 2σ to 3σ | 6 | 0.59% | 22 | 2.14% | -16 | -1.55% |
| >3σ | 2 | 0.20% | 1 | 0.13% | 1 | 0.06% |
| >avg | 575 | 56.99% | 505 | 50.00% | 71 | 6.99% |
| <avg | 422 | 41.82% | 505 | 50.00% | -83 | -8.18% |

The stock market is not normal, but the discrepancies are smaller than most people imagine.

***Notes:***

*The analysis in this report has been prepared by David E. Hultstrom, MBA, CFP©, CFA©.*

*Mr. Hultstrom is the president of Financial Architects, LLC, a financial planning and wealth management firm. Questions or comments are welcome, and he may be reached at David@FinancialArchitectsLLC.com or **(770) 517-8160**.*

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